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AEG5212 Teaching Mathematics 1
Assignment 3 – Pedagogical Content Knowledge Investigation

Decimal Place Value

Decimal place value is a concept that is often difficult for junior secondary school students. It is a time when students are consolidating knowledge about fractions, decimals and percentages as well as attempting to grasp negative numbers, and Irwin (2001) states that understanding decimal fractions is particularly problematic. Goos, Stillman and Vale (2007) show a scenario in which two students were given pairs of decimals numbers and asked to identify the larger value. Neither student accurately identifies the larger values in all cases. Both get some correct and some incorrect. This is because the students are working with a flawed understanding of what constitutes value in decimal numbers.

Student A selects 1.053 as larger than 1.06, 4.08 as larger than 4.7, and 3.073 as larger than 3.72. The pattern that emerges here is that the student believes that the longer the decimal number is, the higher its value. This is known commonly as the 'longer is larger' fallacy. Student B selects 0.4 as larger than 0.457, 0.36 as larger than 0.216, and 5.62 as larger than 5.736. Again a pattern can be identified, this student believes that decimal numbers are larger if they are shorter in length. This is the reverse of Student A, and is another common misconception known as the 'shorter is larger' fallacy. It is clear from this that neither student actually understands decimal place value, their skill level appears to be considerably lacking. Both students are using physical features of the figures (in this case, number of digits) rather than analysing the content of the number and it's meaning. It follows that Student A would identify the larger of 0.276 and 0.29 incorrectly, as they would assume the number with more digits was of higher value. Conversely, Student B would identify the larger of 1.56 and 1.834 incorrectly, because they would assume the shorter number was of greater value.

VELS clearly state that at Level 4 for Number students should "comprehend the size and order of small numbers (to thousandths) and large numbers (to millions)." This implies that students exiting primary school in Year Six should grasp decimal place order. The DEECD Mathematics Continuum discusses the 'longer as larger' and 'shorter as larger' misconceptions that students hold when comparing numbers. Irwin

(2001) also lists several other misconceptions that have been brought to light, such as students believing that adding a zero to the right of a decimal makes it ten times larger, that one hundredths is written 0.100 as well as several other errors that were brought to light in the study undertaken.

It is critical to teach these students to correctly understand decimal place value. The first step would be to introduce them to, or refresh their knowledge of, the decimal place value chart:

Tens	Ones	•	Tenths	Hundredths	Thousandths

Students can enter any decimal numbers they encounter into such a chart to facilitate understanding and comparison. The DEECD Mathematics Continuum lists several activities that can be undertaken to strengthen students' comprehension of place value. One of these is asking students to write down numbers between. For example, students could be set the task of finding twelve numbers that lie between 4.5 and 4.7. This will allow them to explore the range of possible numbers and hopefully solidify the concept that a number such as 4.62 is numerically less than 4.7 even though it contains more digits. Students could also illustrate this on a number line segment in order to assist those who need to refer back to visual representation to assist their understanding.

Irwin's (2001) study investigated the usefulness of everyday context in students' learning to aid comprehension, and found that where context was immediate and relevant the students did perform better. With this in mind, it would be worth exploring real life applications of decimal place order and comparing decimals. For example, most students are familiar with temperatures and dollar values in decimal format. Using these as example exercises might assist some students in anchoring the knowledge to real world contexts.

Teachers should be aware of the common misconceptions, such as the belief that 'longer is larger', in order to ensure they have an accurate overview of student understanding and capabilities. Assessment tasks should be set with these misconceptions in mind, so that any students using patterns rather than content analysis are identified and can be assisted.

Translation Errors

Moving between English or natural language and mathematics and algebraic expressions can pose problems for many students. The process is known as translation, that is taking the word based language problem and converting it into a mathematical expression. Level 4 VELS for Structure state that students should be able to “identify relationships between variables and describe them with language and words (for example, how hunger varies with time of the day).” This means that at the completion of Year Six students should be able to look at an expression and describe it in words. In junior secondary the focus moves to doing the reverse. Goos, Stillman and Vale (2007) reference a series of sentence questions that were posed to a group of students. The students were asked to translate the sentences into algebraic expressions, and the results were evaluated.

The questions are as follows:

4a/b 'The number of animals is equal to ten times the number of zookeepers,' where *a* is number of animal and *z* is number of zookeepers.
Solution: $a = 10\ z$

5a/b 'There are seven times as many toys as children,' where *t* is number of toys and *c* is number of children.
Solution: $t = 7\ c$

6a/b 'For every bus there are twenty passengers,' where *b* is number of buses and *p* is number of passengers.
Solution: $p = 20\ b$

The study found that the question that was solved correctly most often was actually question five. This is an interesting occurrence, because it is not the clearest and most translatable sentence. Padula, Lam and Schmidtke (2001) discuss the issue of syntactic translation, and note that because of the nature of the way many mathematics problems are worded, students are prone to a common error called reversal error. This most often is evidenced by problems that begin with phrases such as 'There are' or 'For every.' It is interesting to note that question four above is the most easily translated directly into algebra, and yet it was not the equation that students performed better on.

It has been concluded that one of the main concerns with translation and the cause of many common student errors is the ingrained understanding of natural language rules or usage of language (Padula et al., 2001). As well as syntax of the sentences, other areas that may affect comprehension are the order that the information is presented in, and the explicitness of the relationships between the variables. Even with this understanding sometimes there are other factors external to syntax and language that aid or impair students' ability to solve problems. In this case it may be that question five was more immediately grounded in their experience, as it related to toys. Or it may have been that the way the question was structured was more familiar to the students from previous experience.

The best way that students are going to learn how to translate word based problems into algebraic expressions is practice. It is only through repetition and exposure that students will become familiar with different types of problems, and the kind of phrases and terms that may be used in such exercises. Purplemath (2011) includes a chart that contains words and phrases commonly found in mathematical word problems and the operations that they relate to. For example, phrases such as 'decreased by', 'minus', 'less', 'difference between/of', 'less than' or 'fewer than' will relate to the operation of subtraction in an algebraic or mathematical expression. It would be beneficial to brainstorm and develop such a chart in class with students, that they can then go on to use as a reference guide.

Repetition and practice will cement the students' experience and ability in terms of language based problems, but careful selection of exercises could assist in acquisition of skills and knowledge. Irwin (2001) talks about the value of context and grounding in everyday experiences for aiding student comprehension. This is relating to decimal place value, but the overall theory holds true for all branches of mathematics – if the students can relate to the problems in real terms, they are more likely to grasp concepts and solve correctly.

There needs to be a balance between devising and using clear, syntactically unambiguous exercises for practice and assessment to assist students, and presenting them with more challenging material. Students will not always be able to rely on problems being well structured, and they should be familiar with strategies to deal with complexity.

Statistical Reasoning

"Statistical thinking offers simple but nonintuitive mental tools for trimming the mass of information, ordering the disordered, separating sense from nonsense, and selecting the relevant few from the irrelevant many." (Ben-Zvi, 2000). In simple terms, statistical thinking and reasoning are critical tools that allow the interpretation of data, and should be deliberately developed. Goos, Stillman and Vale (2007) state that comparing groups is an effective approach to facilitate students' understanding. They provide an example exercise which illustrates the same data collected on spring temperatures in Perth displayed in two different sets of box and whisker graphs. One is total data, one is disaggregated by month.

It is clear that while the representation of the aggregate data for the whole of spring is meaningful and does provide a good amount of useful information, the month by month split allows for more detail. The spring graph shows the median temperature and the range of temperatures that occurred over the entire season, including three extreme high temperature days. The disaggregated graphs make it easy to trace the increasing median temperature over the course of the three months, and also to identify that as the months progress the range of temperatures increases. That information is not readily available from the whole month graph. This is an extremely effective example of how the same data represented in multiple ways can add layers of meaning and richness of interpretation.

Level 5 VELS for Measurement, chance and data sets the standard that:

> Students organise, tabulate and display discrete and continuous data (grouped and ungrouped) using technology for larger data sets. They represent uni-variate data in appropriate graphical forms including dot plots, stem and leaf plots, column graphs, bar charts and histograms. They calculate summary statistics for measures of centre (mean, median, mode) and spread (range, and mean absolute difference), and make simple inferences based on this data.

The concepts and tools required by the standards regarding statistical calculation and visual representation can be used to accomplish students' investigation and comparison of groups. A set of data can be collected by students and represented in a variety of different graphs, plots and charts as listed in the VELS. The students can then analyse these representations and compare what they can interpret from the different forms, and whether some offer more meaning. They may be able to disaggregate data better in some formats than others, or illustrate accompanying calculations such as mean, median or mode in a concise fashion. This exploration of data is referred to by Ben-Zvi (2000) as Exploratory Data Analysis (EDA). EDA is applied in four different stages according to Ben-Zvi: looking at the data, looking between the data, looking beyond the data, and looking behind the data. Ben-Zvi advocates that technological tools make this kind of exploration easier and more accessible to students.

It is important that students develop solid statistical literacy because statistics are used everywhere in life, often in misleading ways. The DEECD Mathematics Continuum includes a range of strategies for assisting students to understand the ways in which data can be displayed and manipulated in order to potentially deceive or mislead. There are activities designed to explore manipulation of scale, and to teach students to consider issues such as sample size and randomness and how that affects the validity of the data.

Statistics and data can be daunting for many students, and so it is important for junior secondary teachers to ensure that their students attain a level of comfort and familiarity in collecting, representing and interpreting data. Like all other areas within mathematics the best way for students to cement understanding is by repeated exposure in many different formats. Spreadsheets and statistical software programs make this simple, but students should also be able to manually work with and visually represent data in a variety of ways.

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