School VU Adult VCE	**Year Group:** Year 11	**Day** Tue	**Date** 8th May 2012
Topic Relations, functions and transformations (2) – Domain and range – Types of relations (including functions)	**Aims** To formally define the concepts of domain and range and explore the use of interval notation in regards to domain and range. To investigate the four different types of relations and to define which types of relations are functions.		**VCE: Mathematics Study Design** Functions and graphs Key knowledge includes: the definition of a function, the concepts of domain and range , and notation for specification of the domain, range and rule of a function Key skills include: specify the rule, domain and range of a relation and identify a relation which is also a function
Location / Setting Classroom D320	**Organisation / Student Groups** Students in normal class groupings		**Classroom management strategy** Standard classroom practice
Key Vocabulary domain, range, ordered pairs, graph, rule, interval, interval notation, independent values, dependent values, functions, vertical line test, one-to-one, one-to-many, many-to-one, many-to-many	**Materials, Resources and Equipment** Overhead visualiser, whiteboard		**References/Sources** Maths Quest 11 – Mathematical Methods CAS Handout

INTRODUCTION Connecting, Engaging and Modelling Inquiry	MAIN BODY Guiding Inquiry and Practise	ASSESSMENT For, as ,of
Warm up: (3 – 5 minutes) Quick review of the previous class. Ask the students if they remember what the two topics covered in the last lesson were (set notation and relations and graphs). Ask a few questions to test students' recall of the material, for example : Is $\pi \in N$? Is data consisting of student height in a class discrete or continuous? **Introduction: (2 – 3 minutes)** Brief discussion of the topics to be covered in the lesson - domain and range, and types of relations and functions	**Domain and range** (5 – 10 minutes) Refer to handout / overhead Go through the definitions of domain and range including the concept of implied domain of a relation. **Interval notation** (5 – 10 minutes) Refer to handout / overhead Go through the eight components of interval notation including the closed/open circle as being inclusive/exclusive of the specified number. **Worked examples** (15 – 20 minutes) Refer to handout / overhead Go through the prepared worked examples on the overhead visualiser / whiteboard as a class. **Types of relations / Functions** (10 – 15 minutes) Refer to handout / overhead Discuss the four different types of relations using the examples provided on the handout., including the definition of a function and the use of the vertical line test in identifying functions. **Worked examples** (15 – 20 minutes) Refer to handout / overhead Go through the prepared worked examples on the overhead visualiser / whiteboard as a class.	**Of** A brief, informal, verbal quiz at the beginning of the lesson will be an assessment of learning regarding material covered in the previous class. **As** Assessment in this class will predominantly be assessment as learning. This will be achieved by attempting to ask a variety of students to assist with answering questions throughout the duration of the class in an attempt to gauge students' comfort levels and understanding. I will also circulate through the class whilst they are working on the text book problems to get an idea of their engagement with and understanding of the material.

	Short break (approx 5 minutes) **Exercises** (35 – 40 minutes) Students to work through exercises 4C on page 181 of the textbook and 4D on page 185 of the textbook.	

Reflection	**Mentor Feedback**
It seemed like the first class lulled me into a false sense of security. Not that this class was a problem, but it did not flow as smoothly as the first, and there was a point where I needed to call the class to attention as the levels of chatter had risen to distracting levels. They did accept and respond to that appropriately. The material was a bit more conceptual, and there were some students who were clearly having difficulties understanding the concept of domain and range. The lesson finished in good time, leaving the students with time to work on exercises. This allowed me to touch base with many of the students in the class and provide assistance where necessary. I ended up sitting with one of the ESL students and helping to guide him through his assignment from the previous chapter. If I had more time I would have like to try incorporating other styles of teaching into my lessons, perhaps using videos or powerpoint presentations, or some other kind of visual aid, or perhaps even something tangible to hold onto. I wonder if some of the students would not have benefited from a more kinaesthetic approach.	

DOMAIN AND RANGE

A relation can be described by:

1. A list of ordered pairs
2. a graph; or
3. a rule.

The *domain* is the set of all first elements of a set of ordered pairs, or the set of independent variables.
The *range* is the set of second elements of a set of ordered pairs, or the set of independent variables.

If a relation is described by a rule, it should also specify the domain. For example:

1. the relation **$\{(x,y): y=2x, x \in \{1,2,3\}\}$** describes the set of ordered pairs **{(1,2),(2,4),(3,6)}**
2. the domain is the set **$X = \{1,2,3\}$**, which is given
3. the range is the set **$Y = \{2,4,6\}$**, and can be found by applying the rule **$y = 2x$** to the domain values.

If the domain of a relation is not specifically stated, it is assumed to consist of all real numbers for which the rule has meaning. This is referred to as the *implied domain* of a relation. For example:

$\{(x,y): y = x^3\}$ has the implied domain R
$\{(x,y): y = \sqrt{x}\}$ has the implied domain $x \geq 0$

Interval Notation

If a and b are real numbers and $a < b$, then the following intervals are defined with an accompanying number line.

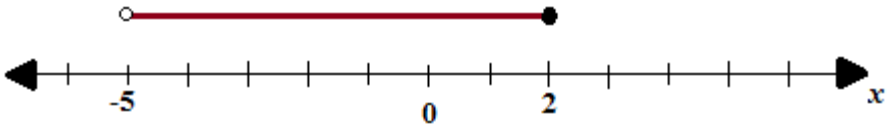
 (a,b) implies $a < x < b$	 $(a,b]$ implies $a < x \leq b$
 (a,∞) implies $x > a$	 $[a,\infty)$ implies $x \geq a$
 $(-\infty,b)$ implies $x < b$	 $(-\infty,b]$ implies $x \leq b$
 $[a,b)$ implies $a \leq x < b$	 $[a,b]$ implies $a \leq x \leq b$

A closed circle indicates that the number is included and an open circle indicates that the number is not included.

Worked examples

1. Describe each of the following subsets of the real numbers using interval notation.

a.



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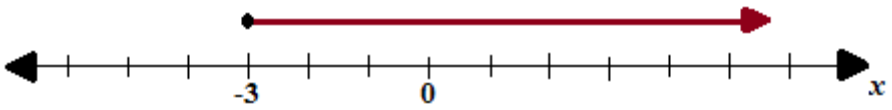

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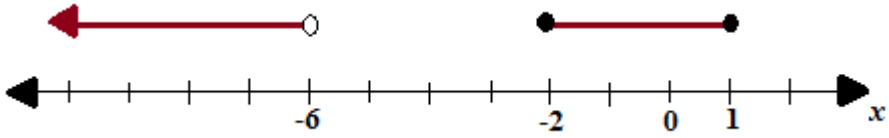

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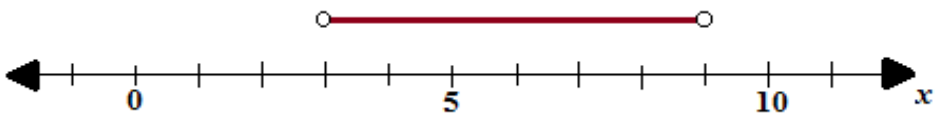

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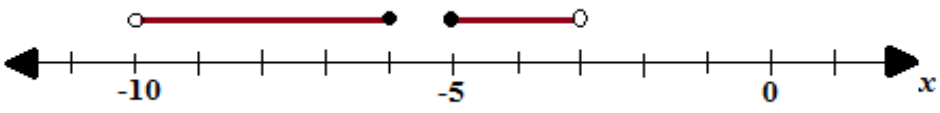

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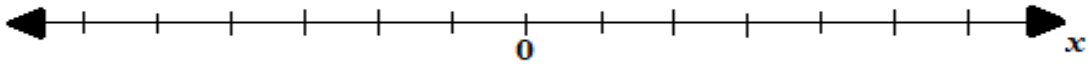
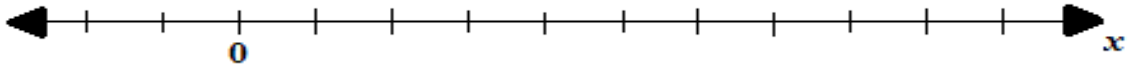
2. Illustrate the following number intervals on a number line.

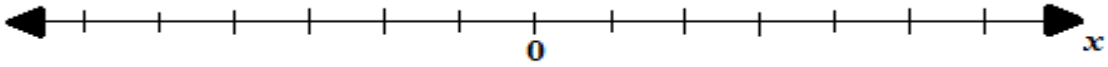
a. **[-4,3)**



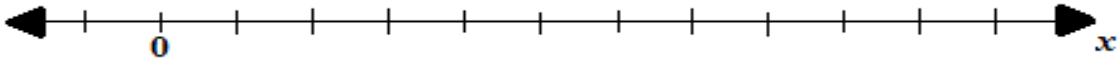
b. **[-1,1) ∪ [4,∞)**



c. **(-∞,0]**



d. **(2,7] ∪ [9,10]**



3. State the domain and range of each of the following relations.

a. {(1,4),(2,7),(3,12),(4,19),(5,28)}

domain _____

range _____

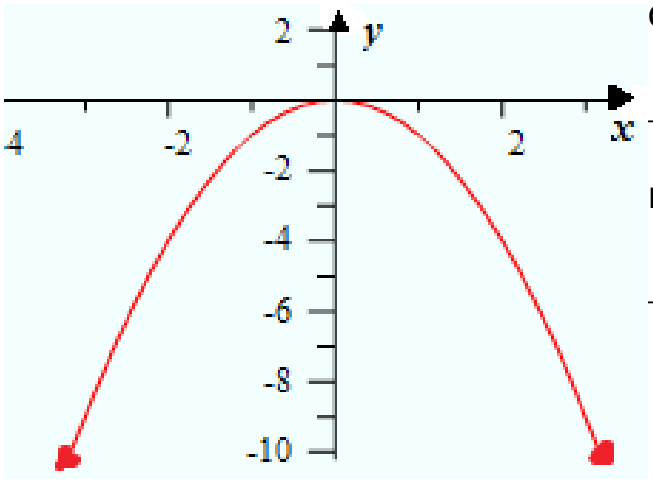
b.

Number of hours of hire	3	4	5	6	7	8
Cost ($)	50	60	70	80	90	100

domain _____

range _____

c.



domain

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range

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d.

domain

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range

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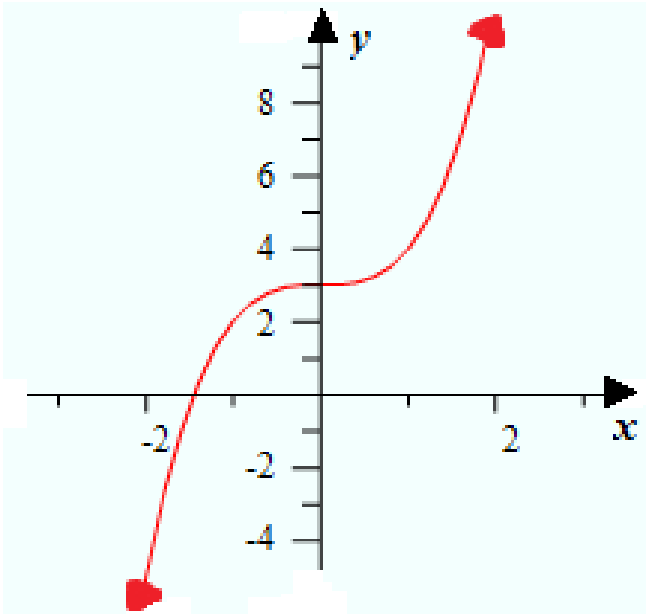

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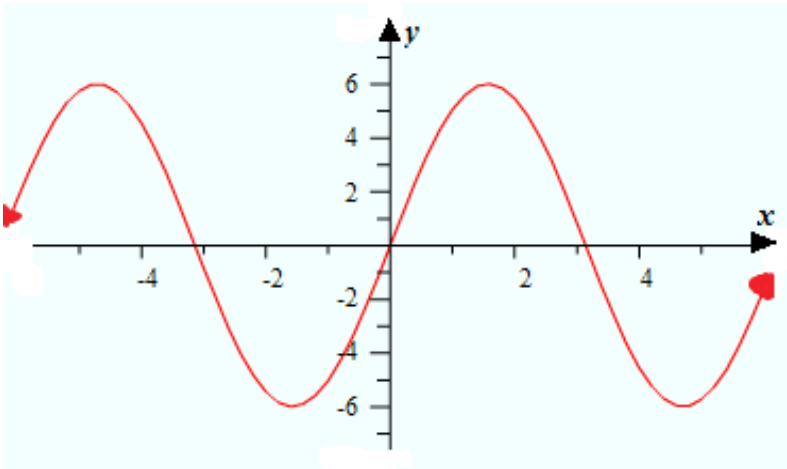

domain _____

range _____

4. For the relation given, sketch its graph and state the domain and range using interval notation.

a. $\{(x,y): y = \sqrt{25-x^2}\}$

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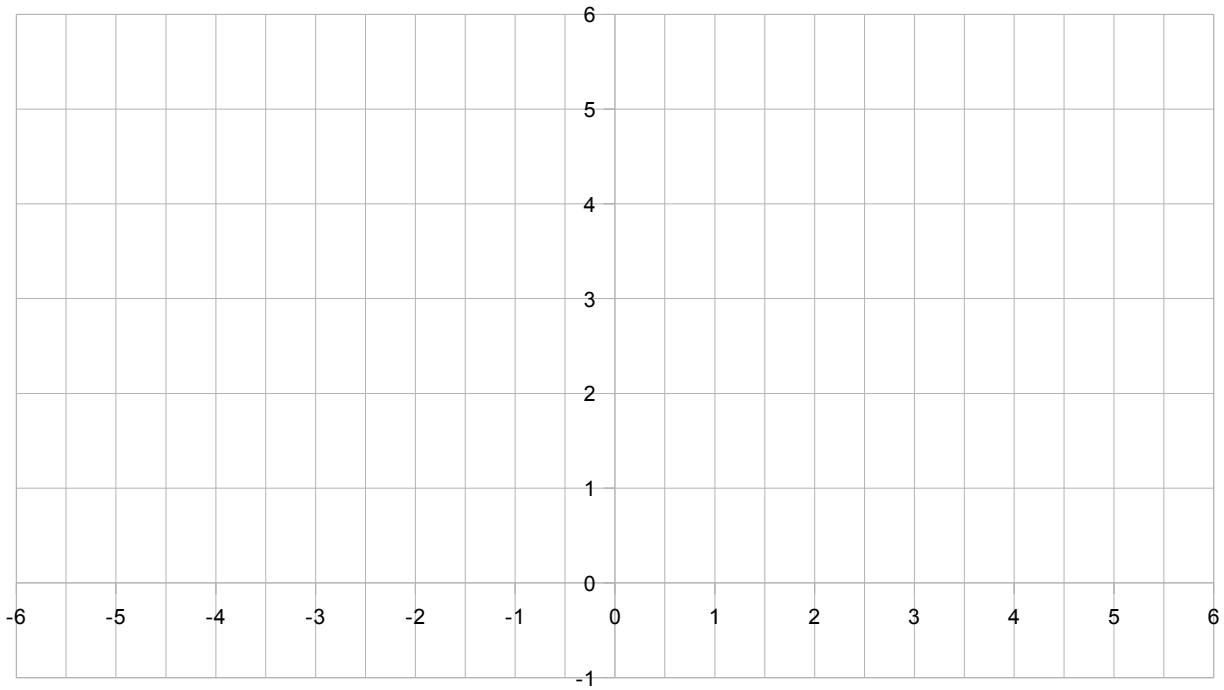
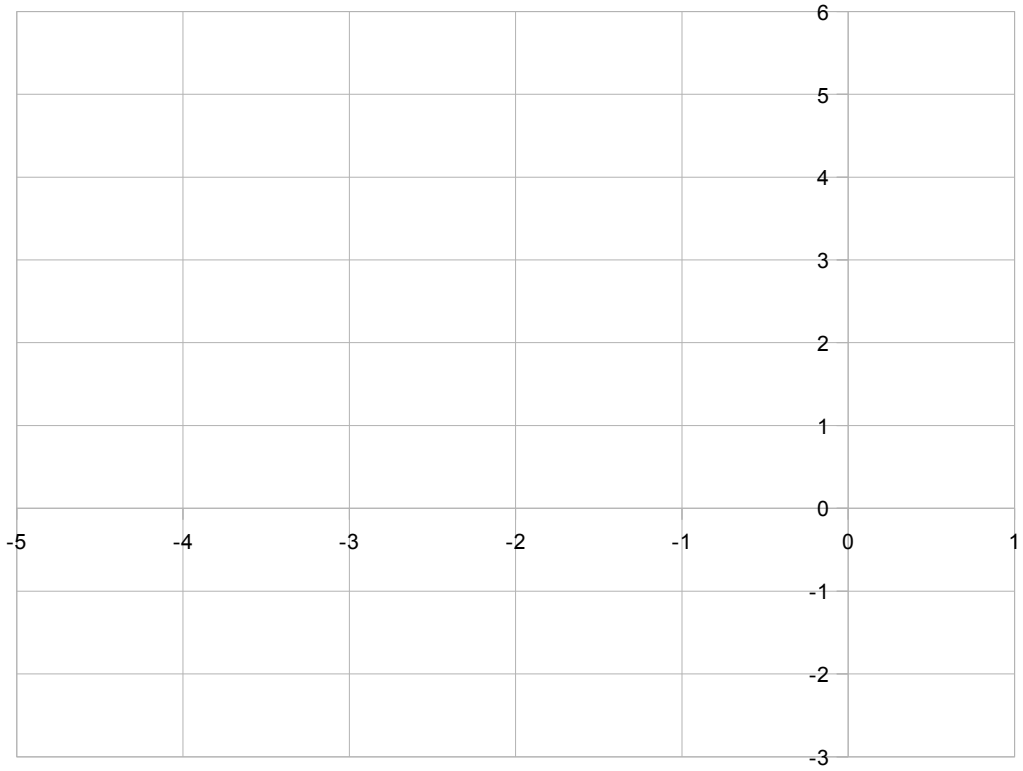


domain _____

range _____

b. {(x,y): $y=\left(\frac{x^3+20}{-10}\right)$, $x \in$ [-4,0]}



domain _____

range _____

c. $\{(x,y): y = x^2 - 4, x \in [-1,3]\}$

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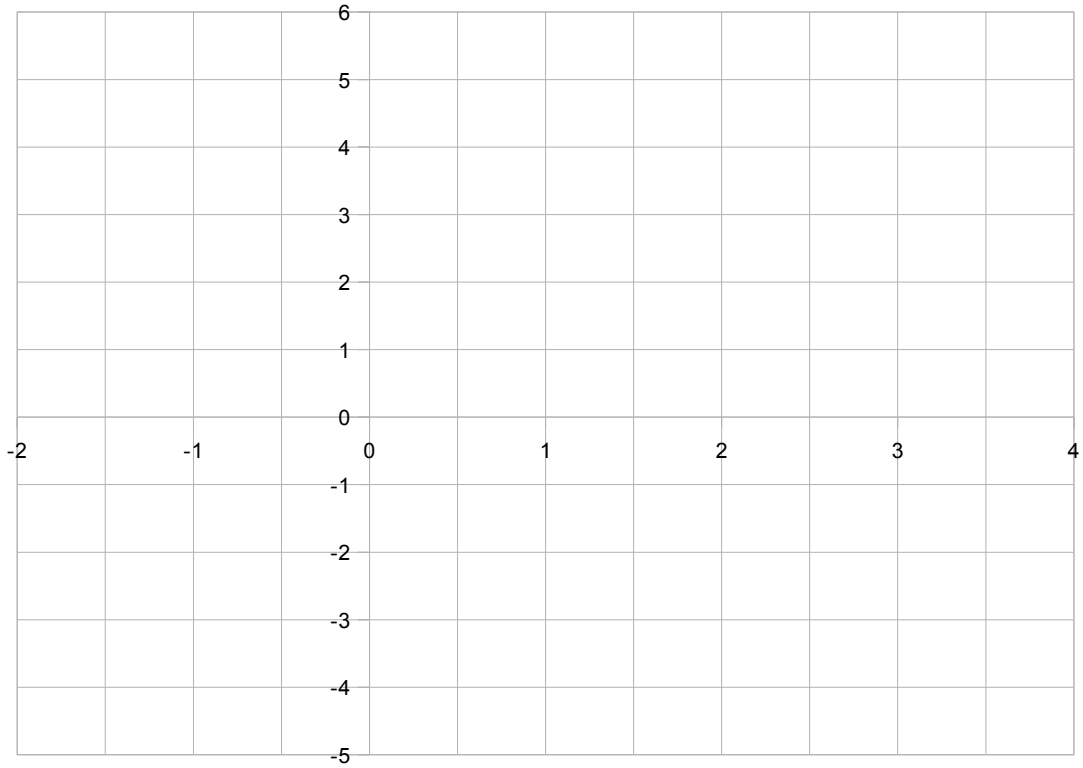


domain _____

range _____

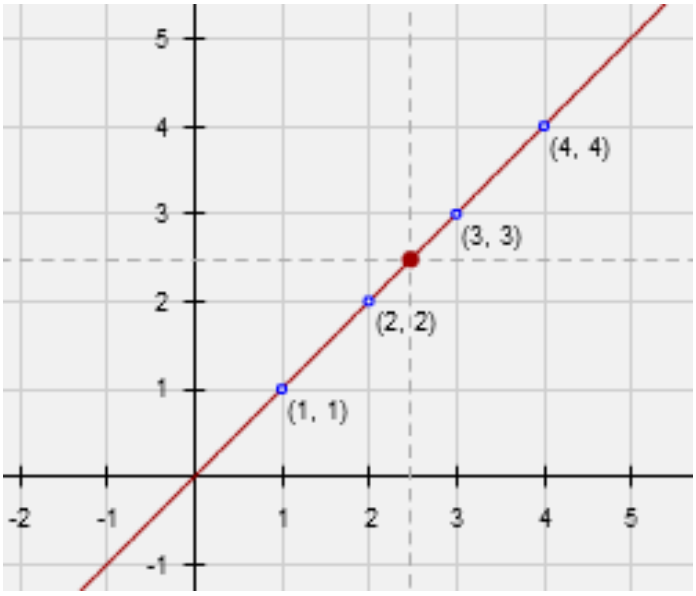
TYPES OF RELATIONS; FUNCTIONS

There are four different types of relations:

1. One-to-one relations
2. One-to-many relations
3. Many-to-one relations
4. Many-to-many relations

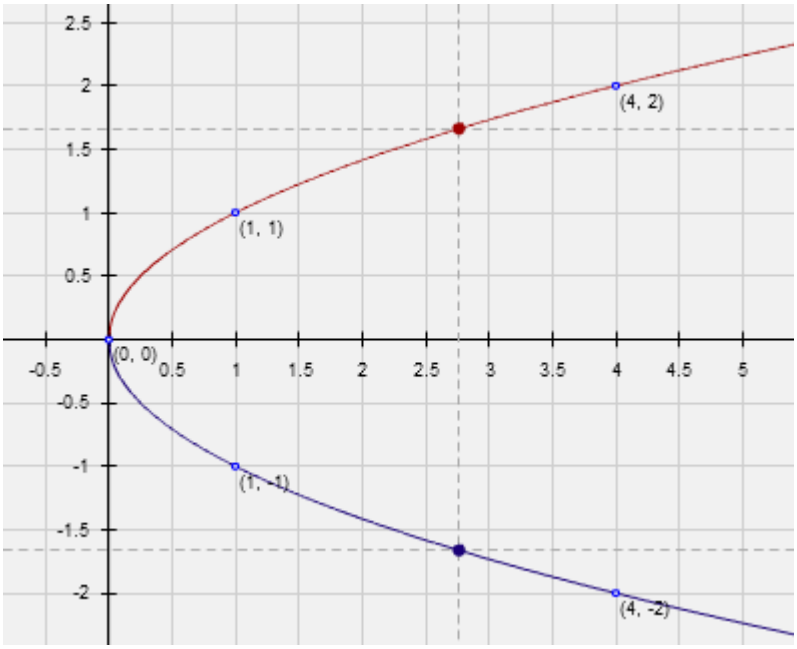
A **one-to-one** relation exists if, for any x-value, there is only one corresponding y-value and vice versa.
For example:

{(1,1),(2,2),(3,3)(4,4)}



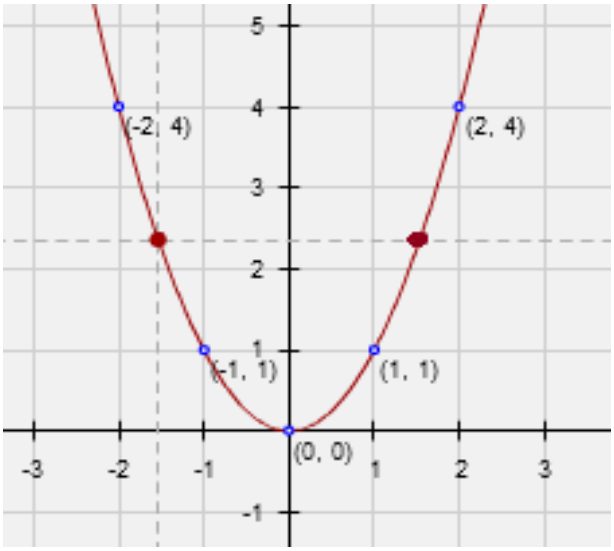
A **one-to-many** relation exists if for any x-value, there is more than one y-value, but for any y-value there is only one x-value.
For example:

{(0,0),(1,1),(1,-1), (4,2),(4,-2)}



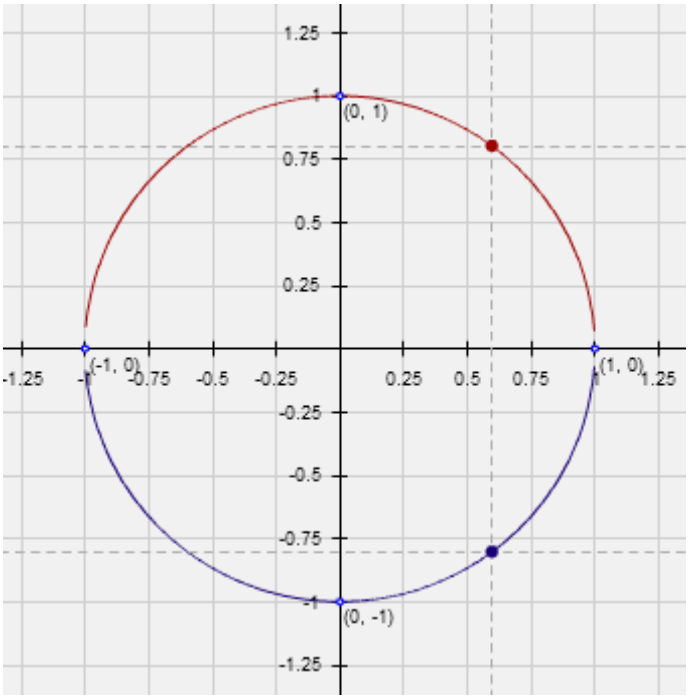
A **many-to-one** relation exists if there is more than one x-value for y-value but for any x-value there is only one y-value.
For example:

{(-2,4),(-1,1),(0,0),(1,1),(2,4)}



A **many-to-many** relation exists if there is more than one x-value for any y-value and vice versa.
For example:

{(-1,0),(0,1),(0,-1),(1,0)}



Relations which are one-to-one or many-to-one are called **functions**. That is, a function is a relation where for any x-value there is at most one y-value.

A function is determined from a graph if a vertical line drawn anywhere on the graph cannot intersect with the curve more than once. This is called the *vertical line test*.

Worked examples

1. What type of relation does each of the following represent? State whether the relations are functions.

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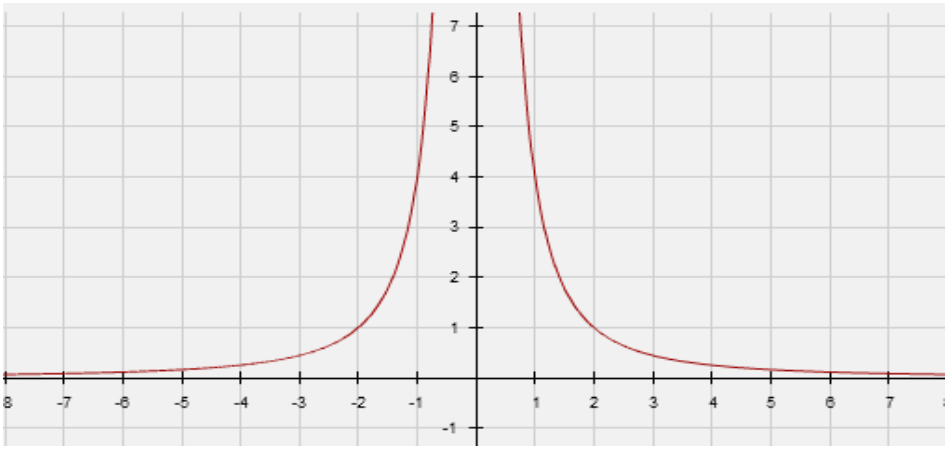

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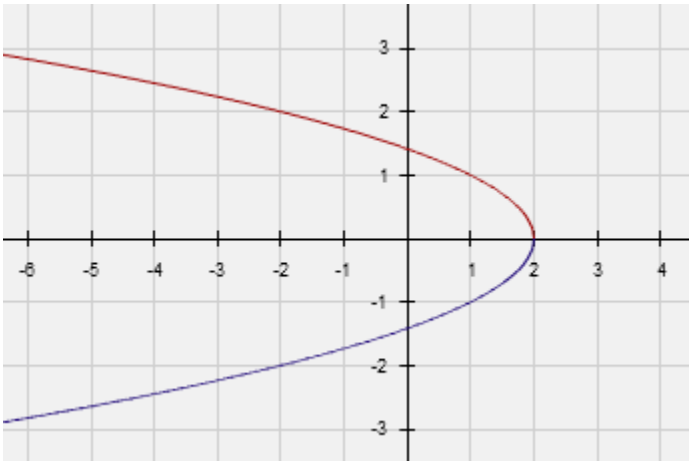

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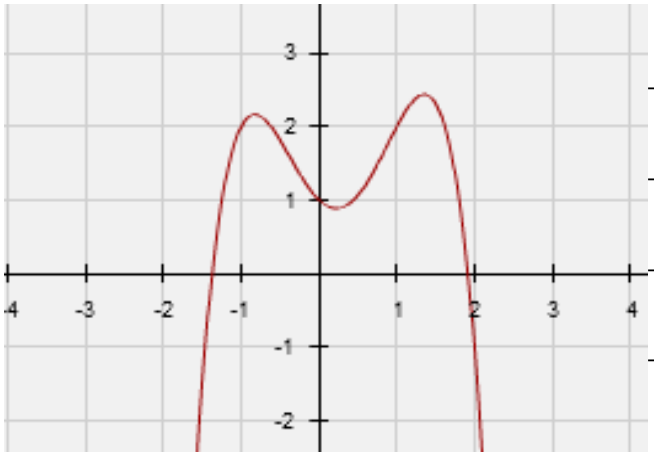

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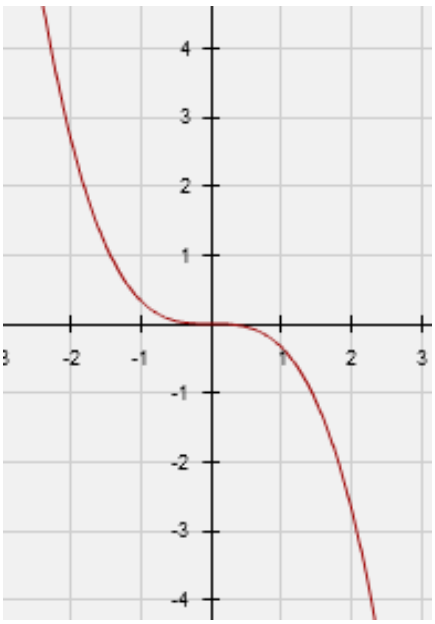
d.



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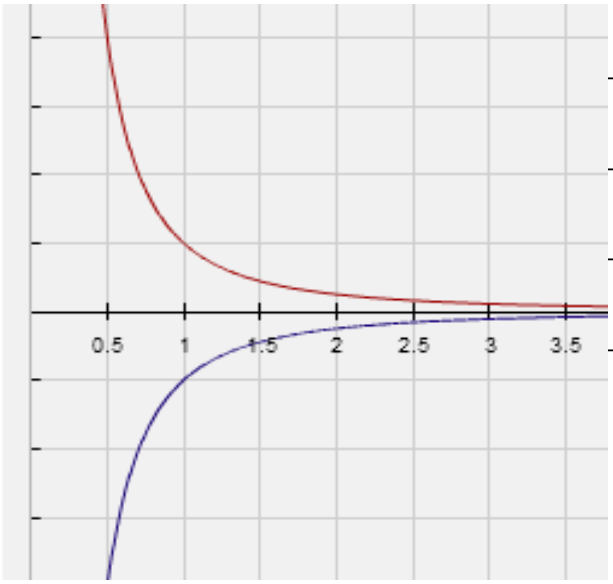

f. **{(-2,5),(-1,3),(0,1),(1,-1)}**